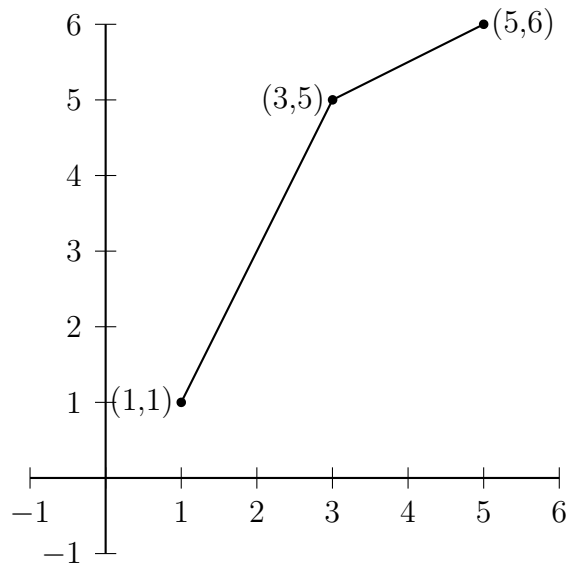
Exercise X

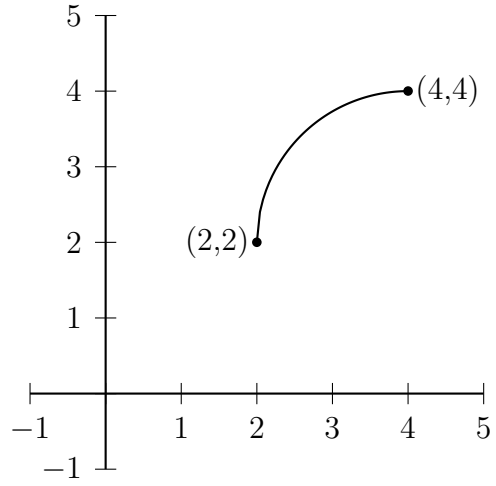
1. Show that $f(x) = x^{\frac{7}{3}}$ is differentiable twice but not three times at 0.

2. The following is the graph of a function $f(x)$:



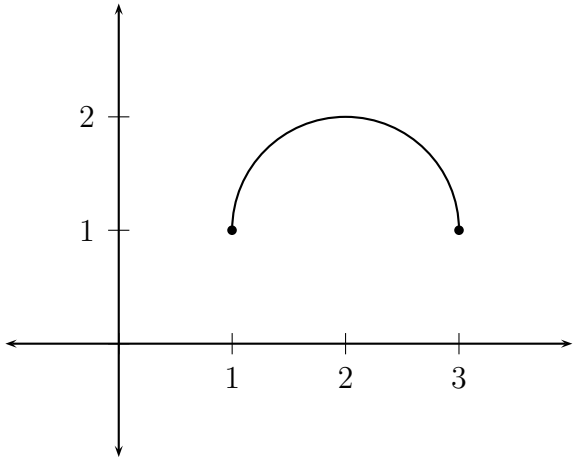
   Sketch the graph of $f^{-1}(x)$.
   Find $\int_1^5 f(x)\mathrm{d}x$.

3. The graph of the function $f(x), x \in [2, 4]$ is a quadrant of a circle with radius 2 as illustrated in the diagram below:



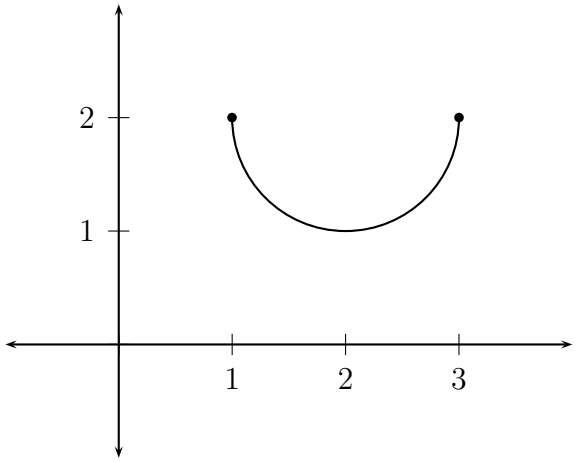
   Sketch the graph of $f^{-1}(x)$.

4. The graph of the function $y = f(x), x \in [1, 3]$ is a semi-circle and is sketched below. The end-points of the graph are $(1, 1)$ and $(3, 1)$:



Find $\int_1^3 f(x)\mathrm{d}x$.

5. The graph of the function $y = f(x), x \in [1, 3]$ is a semi-circle and is sketched below. The end-points of the graph are $(1, 2)$ and $(3, 2)$:



Find $\int_1^3 f(x)\mathrm{d}x$.

6. Determine the area of the hatched region.